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A combinatorial construction for two formulas in Slater's List

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Number 19 in Slater's list [4] is

$$(-q;q)_\infty \sum_{n\geq 0} \frac{(-1)^n q^{3n^2}}{(q^2;q^2)_n(-q;q)_{2n}} = \frac{1}{(q;q^5)_\infty(q^4;q^5)_\infty},$$

where we use the q-Pochhammer symbols

$$(a,q)_n := \prod_{j=1}^{n}(1-aq^{n-1}), \quad (a,q)_\infty = \lim_{n\to\infty}(a,q)_n.$$

We set up a combinatorial framework for inclusion-exclusion on the partitions into distinct parts to obtain the same series as an alternative generating function of partitions into distinct and non-consecutive parts. In connection with Rogers-Ramanujan identities, the generating function yields the aforementioned formula in Slater's list, along with its sister, namely number 15. The same formulas were constructed by Hirschhorn [2]. Similar formulas were obtained by Bringmann, Mahlburg and Nataraj [1]. These are part of the results in [3].

Keywords
integer partition, partition generating function, Rogers-Ramanujan identities, Slater's list

References

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