The category of finite-dimensional representations of periplectic Lie superalgebras

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Let $V = V_{\overline{0}} \oplus V_{\overline{1}}$, a $2n$-dimensional $\mathbf{Z}_2$-graded complex vector space, where $\dim V_{\overline{0}} = \dim V_{\overline{1}} = n$. Then the endomorphism algebra $\mathrm{End}(V)$ inherits the structure of a vector superspace $gl(n|n)$ from V.

Now suppose V is equipped with a nondegenerate odd symmetric form on $V \otimes V$ satisfying

$$\beta(v,w) = \beta(w,v) \quad \text{and} \quad \beta(v,w) = 0 \text{ if } \overline{v} = \overline{w}, \tag{1}$$

where $\overline{v}$ is the parity of a homogeneous element $v \in V$. We define the periplectic (or strange) Lie superalgebra $p(n)$ as the set of all $X \in \mathrm{End}(V)$ preserving the bilinear form β, i.e., X satisfies

$$\beta(Xv,w) + (-1)^{\overline{X}\overline{v}}\beta(v,Xw) = 0. \tag{2}$$

With respect to a fixed basis $V_{\overline{0}} = \mathrm{span}_{\mathbf{C}}\{e_1,\ldots,e_n\}$ and $V_{\overline{1}} = \mathrm{span}_{\mathbf{C}}\{f_1,\ldots,f_n\}$, a periplectic Lie superalgebra is described as

$$p(n) = \left\{ \begin{pmatrix} A & B \\ C & -A^t \end{pmatrix} : B = B^t, C = -C^t \right\}.$$

I will introduce the representation theory of periplectic Lie superalgebras by providing the combinatorics of the category and its underlying highest weight structure, and I will discuss weight diagrams, which are a useful combinatorial tool, allowing us to compute the multiplicities of standard modules in indecomposable projective modules and of simple modules in standard modules.

More precisely, translation functors on the category $\mathscr{F}_n$ of finite-dimensional representations of $p(n)$ using the endomorphism of the endofunctor $-\otimes V$ will be defined. I will then define the actions of translation functors on thick and thin Kac modules, which categorically lift the Temperley-Lieb relations associated to the infinite symmetric group. Next, I will define the notion of weight diagrams for dominant weights and explain the associated combinatorics of the actions of translation functors on standard and costandard objects in terms of the diagrams. This involves moving a shaded ball left or right, depending on the translation functor and the weight. I will also explain the duality for simple modules in terms of weight diagrams.

Finally, we define the minimal equivalence relation on the set of dominant weights λ and μ such that $\lambda \sim \mu$ if μ is obtained from λ by sliding a shaded ball in a certain way. This implies that simple modules $L(\lambda)$ and $L(\mu)$ belong to the same block if and only if $\lambda \sim \mu$. I will give a classification of the blocks in $\mathscr{F}_n$ and describe the action of translation functors on these blocks.

References

[1] M. Balagovic, Z. Daugherty, I. Entova-Aizenbud, I. Halacheva, J. Hennig, M. Im, G. Letzter, E. Norton, V. Serganova, and C. Stroppel, *Translation functors and decomposition numbers for the periplectic Lie superalgebra $p(n)$,* submitted, arXiv:1610.08470.